sandwich construction, cylindrical shell, elastic buckling, FEM

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BUCKLING OF SANDWICH CYLINDRICAL SHELL WITH CORRUGATED MAIN CORE AND THREE-LAYER FACES

Abstract

The subject of numerical research is a seven-layer cylindrical shell subjected uniformly distributed external pressure. The shell is thin-walled sandwich structure composed of main corrugated core (made of a thin metal sheet) and two three-layer faces. The cores of the faces are porous and made of isotropic metal foam. The corrugation of the main core is along longitudinal axis of the shell. The shell is simply supported at its all outer edges. Numerical FEM model of the shell is elaborated. Critical pressure for the family of these shells are calculated. Furthermore, developed a model of an equivalent single-layer shell wherein diameter, and weight are the same as the seven-layer shell. It has been shown several times higher resistance to buckling of a seven-layer shell compared to the single-layer shell.

1. INTRODUCTION

Thin-walled structures (beams, plates and shell) are heavily used in many branches of the industry. This elements are used as basic structural parts in simple and complex structural systems – structures of aircrafts and rockets, marine vehicles, tanks for liquids and gases, building structures and many others. For the construction of thin-walled components are used various materials, for example: metals, composites, polyurethanes or metal foams (used as cores for sandwich structure). Moreover, the structure of thin-walled components may be: homogeneous, sandwich, multilayered as well as inhomogeneous continuous (continuous structure with varying density). A large variety of thin-walled structures is caused by their properties and the intended use.

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Influence the choice of an appropriate structure are the following factors: the use of construction, load type of structure, load bearing capacity, vibration damping, cost of production and operation of the equipment, insulation performance, technology of production, etc. This paper focused on a sandwich structure.

Classical sandwich structure is composed of two thin metal or composite laminated faces connected with a relatively thick soft core made of foam or low strength of a suitably shaped sheet (waves, honeycomb, etc.). The main assessment criterion of the practical application efficiency of these structures, except for the economic aspect, is a high stiffness and strength with respect to low weight and the high ability of vibration damping. Due to exceptional properties, sandwich structures have been used in aircraft, marine vehicles, and other types of structures. Sandwich structures have been extensively investigated. The first theoretical models of these structure have developed C. Libove and S. B. Butdorf for sandwich plates with corrugated cores, and E. Reissner for sandwich rectangular plates in 1948 [1, 2]. In the following years, rapid growth of works describing these structures is noticeable, e.g. monographs [3, 4, 5]. In these works are presented the research of strength and stability problems of classical sandwich beams, plates, and shells with homogeneous core. These structures also nowadays are intensively developed. Analytical and numerical investigation of strength and stability of an isotropic metal foam beams, plates and shells are presented in the papers [6, 7, 8, 9, 10]. In these works, sandwich and continuous structure with varying density were analyzed. Experimental studies of the sandwich cylindrical shell with a core of the truss structure, connected to the composite facings are presented in the paper [11].

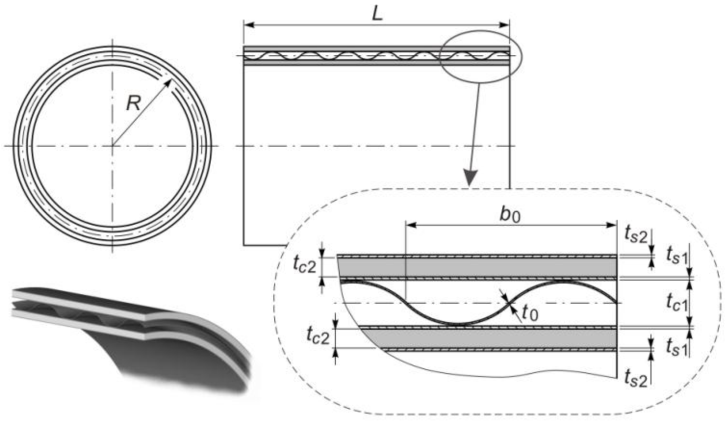

Fig. 1. The cross section of the seven-layer cylindrical shell [source: own study]

Also sandwich structures with corrugated core were studied, e.g. the three-layer beam with a crosswise and lengthwise corrugated core [12], the seven-layer circular plate with a corrugated main core and three-layer faces [13]. Presented here, are just a few of works devoted to a specific issues. A lot of research (theoretical and experimental) dedicated to thin-walled structures indicates that the issues are still relevant.

The subject of this paper is a seven-layer cylindrical shell subjected to uniformly distributed external pressure. The shell is simply supported at its all outer edges. The wall of the shell is composed of a corrugated main core – made of a thin metal sheet, and two three-layer facings with metal foam cores (Fig. 1). The numerical FEM analyses for elastic buckling of the shell are presented.

2. FEM MODEL OF THE MULTILAYERED SHELL

Stability analysis of a seven-layer cylindrical shell using the FEM is carried out with the use of ANSYS software. The edges of the shell are simply supported and closed by thin rings connecting the different layers (Fig. 2). The boundary conditions for the shell were defined on the two outer edges and the middle surface of the main core of the shell. On the outer edges, the radial and the circumferential displacements are blocked. In the middle length of the shell the longitudinal displacements were blocked. The uniformly distributed pressure was added on the outer surface (layer of a thickness t_{s2}). Rings placed on outer edges of the shell are needed for structural reasons, e.g., due to a connection of the cylindrical shell with corrugated core.

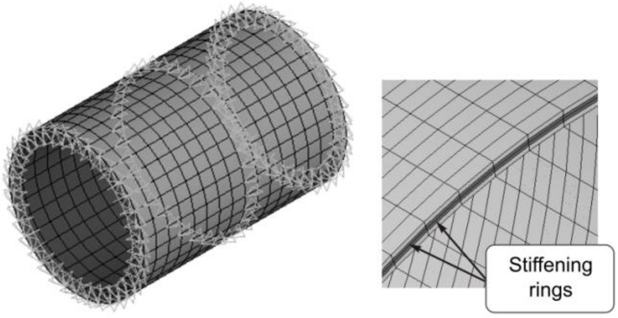

Fig. 2. Model of the shell [source: own study]

The numerical model of the shell was developed in the cylindrical coordinate system. Thin layers (sheets of thicknesses t_{s1} and t_{s2}, corrugated sheet of the core of a thickness t_0 and stiffening rings edges of the shell) were modeled thin-walled elements SHELL 181, whereas the cores of faces, made of metal foam of a thickness t_{c2}, were modeled solid elements SOLID 185. The model of the shell consisted of ca. 30 000 finite elements on 1 m length of the shell.

3. MODEL OF EQUIVALENT SINGLE-LAYER SHELL

The seven-layer shell will be compared with the single-layer shell. For this purpose, a model of the equivalent single-layer shell was developed, wherein radius of the middle surface R, and weight are the same as the seven-layer shell. The unit weight of the seven-layer shell

$$\tilde{m}_{shell}^{(7-layer)} = \left[2t_{s1} + 2t_{s2} + t_{c1,eq} + 2t_{c2}\frac{\rho_{c2}}{\rho_s}\right]\rho_s \qquad (1)$$

where: ρ_s = 7850 kg/m^3 – mass density of the steel,
ρ_{c2} = 0.145ρ_s – mass density of the metal foam [14],
$t_{c1,eq}$ – the equivalent thickness of the corrugated core,

$$t_{c1,eq} = 4\frac{t_0 R_{c1}}{b_0}\,\mathrm{arctg}\left[\frac{4b_0(t_{c1}-t_0)}{b_0^2 - 4(t_{c1}-t_0)^2}\right] \qquad (2)$$

where: R_{c1} – the radius of the middle surface of the corrugation (Fig. 3)

$$R_{c1} = \frac{b_0^2 + 4(t_{c1}-t_0)^2}{16(t_{c1}-t_0)} \qquad (3)$$

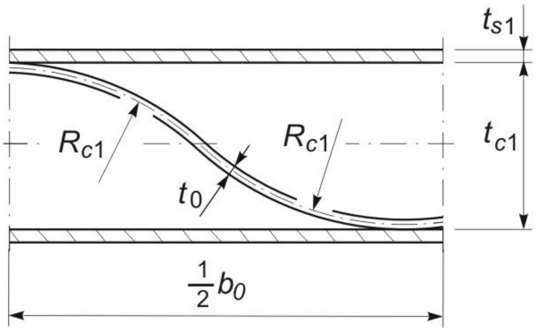

Fig. 3. The main core of the shell [source: own study]

The thickness of the equivalent single-layer shell

$$t_1 = \frac{\tilde{m}_{shell}^{(7-layer)}}{\rho_s} \qquad (4)$$

4. NUMERICAL CALCULATIONS – FEM ANALYSIS

The critical pressure is calculated for the family of seven-layer cylindrical shells: material properties: $E_s = 200000$ MPa, $\nu_s = 0.3$ (the steel elements) and $E_{c2} = 3150$ MPa, $\nu_{c2} = 0.05$ (the metal foam of the cores); geometric data: $R = 2$ m, $L = 2 – 4$ m, $t_0 = 0.6$ mm, $t_{s1} = 0.8$ mm, $t_{s2} = 1$ mm, $t_{c1} = 20$ mm, $t_{c2} = 15$ mm, $b_0 = 100 – 200$ mm. The results of elastic buckling modes of selected multilayered shells and equivalent single-layer shells with the thickness t_1 (Eq. 4) are shown in Figs. 4 – 6. The influence of width of unit corrugation on critical pressure for seven-layer shell presents Fig. 7. The values of critical pressure of seven-layer shell indicate that the critical pressure depends on width of unit corrugation. The value of critical pressure is bigger for smaller values of b_0.

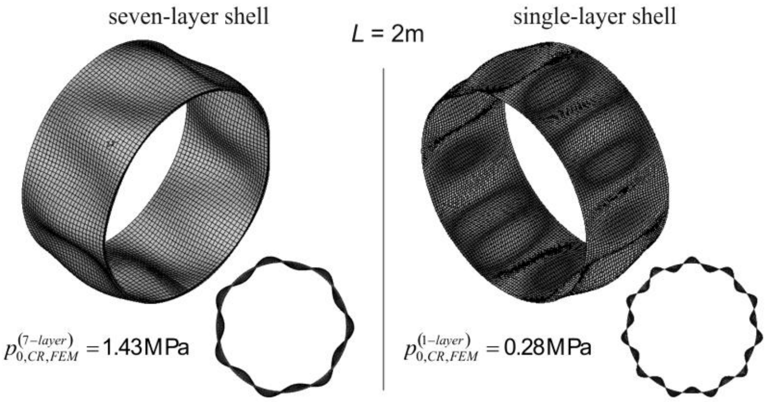

Fig. 4. Buckling mode of seven-layer and equivalent single-layer shells, L=2m, b_0=125mm [source: own study]

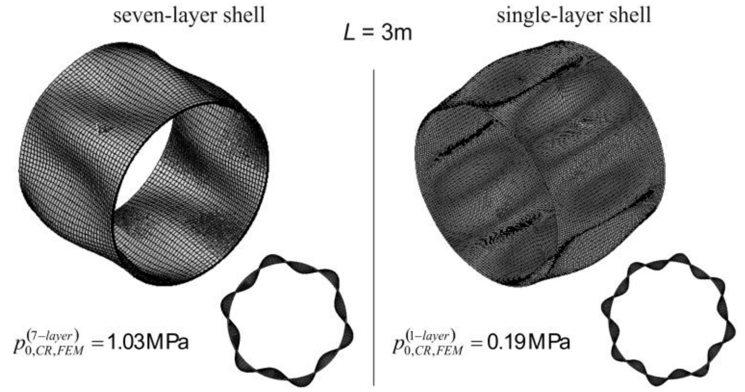

Fig. 5. Buckling mode of seven-layer and equivalent single-layer shells, L=3m, b_0=125mm [source: own study]

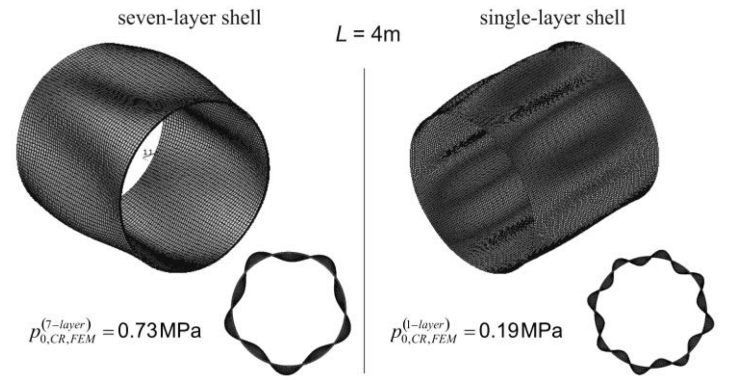

Fig. 6. Buckling mode of seven-layer and equivalent single-layer shells, L=4m, b_0=125mm [source: own study]

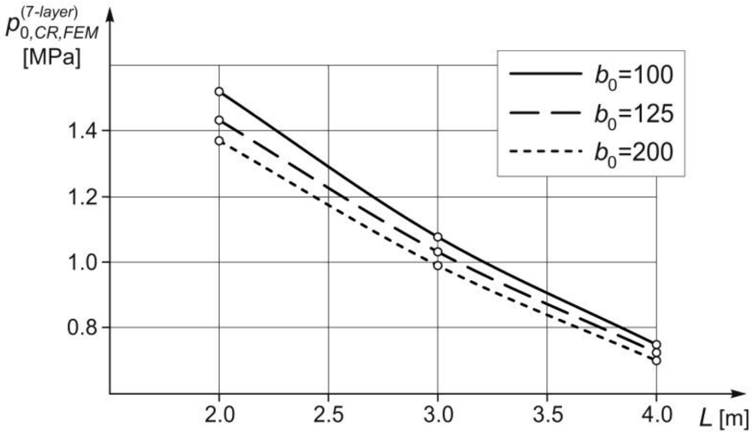

Fig. 7. The effect of the length L on critical pressure multilayered cylindrical shell. Results are plotted for three different sets of width of unit corrugation b_0 [mm] [source: own study]

These results of critical external pressure are presented in Table 1 for different values of the length L and width of unit corrugation b_0. The obtained results have been compared with one-layer shells. The seven-layer shells have smaller values of number of circumferential semi-waves compared with the one-layer shell with equivalent thickness. For example, the seven-layer shell (L = 2m, b = 100mm) has the number of circumferential semi-waves equal 12 and equivalent single-layer shell has m = 22. The seven-layer cylindrical shell is more rigid then single-layer shell of then same weight. The parameter k_p denotes the ratio of the critical pressure of the seven and single layer shells. The value of k_p is greater than one. The values of k_p ratio are in the range 4.9 – 6.3.

Tab. 1. Critical pressure and numbers of longitudinal and circumferential waves

L [m]	b_0 [mm]	7-layer shell			1-layer shell				k_p
		$p_{0,cr,FEM}$ [MPa]	m	n	t_1 [mm]	$p_{0,cr,FEM}$ [MPa]	m	n	
2	100	1.518	1	12	8,61	0.284	1	22	5.3
	125	1.433	1	14	8,59	0.283	1	22	5.1
	200	1.369	1	28	8,57	0.281	1	22	4.9
3	100	1.076	1	12	8,61	0.189	1	18	5.7
	125	1.033	1	12	8,59	0.188	1	18	5.5
	200	0.989	1	20	8,57	0.187	1	18	5.3
4	100	0.749	1	10	8,61	0.119	1	16	6.3
	125	0.726	1	10	8,59	0.118	1	16	6.1
	200	0.700	1	16	8,57	0.117	1	16	6.0

5. CONCLUSIONS

In the paper the numerical model of simply supported sandwich (seven-layer) cylindrical shells under external pressure are presented. Numerical calculations on a family of seven-layer shells have been performed with the use of FE method (ANSYS). Additionally for single-layer shell with equivalent thickness have been carried out. In both cases a radius of the middle surface R, and weight are the same. The values of critical pressure obtained from each sandwich shell's model is higher from 4.9 to 6.3 times then equivalent single-layer shells. As to the externally pressurised sandwich shells a influence of the width of unit corrugation b_0 on the critical pressure is observed (Fig. 7). The higher number of corrugations into main core stiffens more and shell has higher resistance to buckling, and the critical pressure increases.

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