

*enhanced numerical methods for gear life evaluation,
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THE ACCURACY OF AN ACCELERATED METHOD FOR THE EVALUATION OF LIFE OF CYLINDRICAL GEARS WITH PROFILE CORRECTION

Abstract

This paper presents an accelerated method for the evaluation of life of cylindrical gears with profile correction. Compared to a more accurate solution based on the generalized method for evaluating the wear and life of cylindrical involute gears, the new method enables accelerating this process by over 16000 times. The method was used to estimate the life of a cylindrical spur gear depending on different ranges of blocks describing gear teeth meshing conditions (i.e. the number of gear revolutions). The accuracy of the results obtained thereby was examined, too. It has been found that when the range of a block is increased by 3000 times, the deviations of the numerical solution are below 0.5%.

1. INTRODUCTION

When designing gears, it is important from a practical point of view that their life be estimated along with their constructional parameters (including profile correction), operating parameters, wear and meshing conditions. The few evaluation methods which are reported in the literature [1, 2] are not suitable for this purpose. The estimation of wear and life of cylindrical involute gears is, however, possible using the new evaluation method [3–5] which was developed i.a. by the author of this paper.

Depending on service conditions, gears reach their allowable wear after a certain number of revolutions of the rack, ranging from hundreds of million up to several billion revolutions. As a result, the application of the generalized method to estimate gear life, including tooth wear per every revolution of the rack, will require a significantly long computation time, too. In order to shorten the

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computation time to a significant extent, we every revolution of the rack, will require a significantly long computation time, too. In order to shorten the computation time to a significant extent, we developed an enhanced, accelerated method. Below, we present the results of gear life estimation by this method as well as the results concerning the accuracy of the method itself.

2. METHOD FOR ESTIMATING GEAR LIFE

As a result of gear teeth wear, the curvature radii ρ_{jh} of the their profiles increase; this, in turn, causes the initial maximum contact stresses p_{jhmax} to decrease, while the width of the contact areas $2b_{jh}$ increases. The contact stresses at wear are determined by the Hertz equations:

$$p_{jhmax} = 0,564\sqrt{N'/\theta\rho_{jh}}, \quad 2b_{jh} = 2,256\sqrt{\theta N'\rho_{jh}}, \quad (1)$$

where $j = 0, 1, 2, 3, \dots, s$ are the contact points on the active face of the gear teeth; $j = 0, j = s$ are the first and last points of gear teeth engagement, respectively; $N' = N/l_{\min}w$; $N = T_{nom}K_g/r_{w1}\cos\alpha_w$ is the force acting in the engagement; $T_{nom} = 9550P/n_1$ is the rated torque; r_{w1} is the rolling radius of the pinion; α_w is the pressure angle of the corrected profile; P is the power on the drive shaft (pinion); n_1 is the number of revolutions of the drive shaft; K_g is the dynamic factor; $\theta = (1-\mu_1^2)/E_1 + (1-\mu_2^2)/E_2$; E, μ are the Young's modulus and the Poisson's ratio of the materials of gear teeth, respectively; l_{\min} is the minimum length of the contact line; w is the number of engaged tooth pairs; $\rho_{jh} = \frac{\rho_{1jh}\rho_{2jh}}{\rho_{1jh} + \rho_{2jh}}$ is the reduced radius of curvature of the gear profile subjected to change due to wear, in a normal section; ρ_{1jh}, ρ_{jh} are the changeable radii of curvature of the pinion and gear tooth profiles, respectively.

The initial reduced radius of the curvature ρ_j of a cylindrical gear profile is:

$$\rho_j = \frac{\rho_{1j}\rho_{2j}}{\rho_{1j} + \rho_{2j}}, \quad (2)$$

where: ρ_{1j}, ρ_{2j} are the radii of curvature of the flank of unworn profiles of the pinion and the gear, respectively.

The formulas for calculating the radii of curvature of the corrected pinion and gear profiles of the cylindrical gear at j -th point of contact are [3–5]:

$$\rho_{1j} = \frac{\rho_{t1j}}{\cos \beta_b}, \quad \rho_{2j} = \frac{\rho_{t2j}}{\cos \beta_b}, \quad (3)$$

where: $\beta_b = \arcsin(\operatorname{tg} \beta \cos \alpha_t)$, $\alpha_t = \operatorname{arctg} \left(\frac{\operatorname{tg} \alpha}{\cos \beta} \right)$,

$$\rho_{t1j} = r_{b1} \operatorname{tg} \alpha_{t1j}, \quad \rho_{t2j} = r_{w2} \sqrt{\left(r_{2j} / r_{w2} \right)^2 - \cos^2 \alpha_w},$$

$$\alpha_{t1j} = \operatorname{arctg} \left(\operatorname{tg} \alpha_{t10} + j \Delta \varphi \right), \quad \alpha_{t1s} = \operatorname{arctg} \sqrt{\left(r_{1s} / r_{w1} \right)^2 - \cos^2 \alpha_w},$$

$$\alpha_{t2j} = \arccos \left[\left(r_{2w} / r_{2j} \right) \cos \alpha_w \right],$$

$$r_{b1} = r_1 \cos \alpha_t, \quad r_1 = m z_1 / 2 \cos \beta, \quad r_{b2} = r_2 \cos \alpha_t, \quad r_2 = m z_2 / 2 \cos \beta,$$

$$\operatorname{tg} \alpha_{t10} = (1 + u) \operatorname{tg} \alpha_w - \frac{u}{\cos \alpha_w} \sqrt{\left(r_{20} / r_{w2} \right)^2 - \cos^2 \alpha_w}, \quad r_{a2} = r_2 + m,$$

$$r_{20} = r_{a2} - r, \quad r = 0,2m, \quad r_{1j} = r_{w1} \cos \alpha_w / \cos \alpha_{t1j},$$

$$r_{2j} = \sqrt{a_w^2 + r_{1j}^2 - 2a_w r_{1j} \cos(\alpha_w - \alpha_{t1j})}, \quad a_w = (z_1 + z_2) m / 2 \cos \beta;$$

where: r_1, r_2 are the radii of pitch circles of the pinion and the gear, respectively; r_{b1}, r_{b2} are the radii of base circles of the pinion and the gear, respectively; r_{a1}, r_{a2} are the radii of addendum circles of the pinion and the gear, respectively; r is the radius of the gear tooth fillet; u - is the gear ratio; $\Delta \varphi$ is the selected angle of revolution of the pinion from the point of initial contact (point 0) to point 1, and so on; $\alpha = 20^\circ$ is the angle of engagement; β is the pitch angle; m is the engagement module; z_1, z_2 denote the number of gear teeth; α_{10} is the angle of the first point on the contact line; α_{1s} is the angle indicating the location of the last point of engagement of the pinion teeth on the contact line; α_{20}, α_{2s} are the angles indicating the location of the first and last points of engagement of the gear teeth on the contact line.

The minimum length of the contact line is:

$$l_{\min} = \frac{b_w \varepsilon_\alpha}{\cos \beta_b} \left[1 - \frac{(1-n_\alpha)(1-n_\beta)}{\varepsilon_\alpha \varepsilon_\beta} \right] \text{ at } n_\alpha + n_\beta > 1, \quad (4)$$

$$l_{\min} = \frac{b_w \varepsilon_\alpha}{\cos \beta_b} \left[1 - \frac{n_\alpha n_\beta}{\varepsilon_\alpha \varepsilon_\beta} \right] \text{ at } n_\alpha + n_\beta \leq 1,$$

where: b_w is the width of the pinion; $\varepsilon_\alpha, \varepsilon_\beta$ are the coefficients describing the top and step-by-step overlaps of the gear; n_α, n_β are the fractional parts of the

coefficients $\varepsilon_\alpha, \varepsilon_\beta$; $\varepsilon_\alpha = \frac{t_1 + t_2}{t_z}$, $\varepsilon_\beta = \frac{b_w \sin \beta}{\pi m}$, $\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$,

$$t_1 = \frac{e_1}{\omega_1 r_{b1}}, \quad t_2 = \frac{e_2}{\omega_1 r_{b1}}, \quad t_z = \frac{2\pi}{z_1 \omega_1}, \quad e_1 = \sqrt{r_{1s}^2 - r_{b1}^2} - r_1 \sin \alpha_t,$$

$e_2 = \sqrt{r_{20}^2 - r_{b2}^2} - r_2 \sin \alpha_t$, $r_{1s} = r_{a1} - r$, $r_{a1} = r_1 + m$; ω_1 is the angular velocity of the pinion.

In addition, we must include the variations in the radii of gear teeth tips along with the profile correction:

$$r_{a1} = r_1 + (1 + x_1)m, \quad r_{a2} = r_2 + (1 + x_2)m \quad (5)$$

where: $x_1 = -x_2$ are the addendum correction coefficients; for this kind of profile correction: $r_{w1} = r_1, r_{w2} = r_2, \alpha_w = \alpha_t$.

The angles of transition from a double tooth engagement ($\Delta\varphi_{1F_2}$) to a single tooth engagement and, again, to a double tooth engagement ($\Delta\varphi_{1F_1}$) in the profile-corrected cylindrical helical gear are determined from:

$$\Delta\varphi_{1F_2} = \varphi_{10} - \varphi_{1F_2}, \quad \Delta\varphi_{1F_1} = \varphi_{10} + \varphi_{1F_1} \quad (6)$$

where $\varphi_{1F_2} = \tan \alpha_{F_2} - \tan \alpha_w$, $\varphi_{1F_1} = \tan \alpha_{F_1} - \tan \alpha_w$, $\varphi_{10} = \tan \alpha_{r10} - \tan \alpha_w$;

$$\tan \alpha_{F_2} = \frac{r_{w1} \sin \alpha_w - (p_b - e_1) + 0.5n_\beta p_b}{r_1 \cos \alpha}, \quad \tan \alpha_{F_1} = \frac{r_{w1} \sin \alpha_w - (p_b - e_2) - 0.5n_\beta p_b}{r_1 \cos \alpha};$$

$p_b = \pi m \cos \alpha_w / \cos \beta$ denotes the tooth pitch; $e_1 = \sqrt{r_{1s}^2 - r_{b1}^2} - r_{w1} \sin \alpha_w$,

$$e_2 = \sqrt{r_{20}^2 - r_{b2}^2} - r_{w2} \sin \alpha_w.$$

The angles $\Delta\varphi_{1E}$ of teeth exit from the mesh are determined similarly to the above method, namely:

$$\Delta\varphi_{1E} = \varphi_{10} + \varphi_{1E} \quad (7)$$

where: $\varphi_{1E} = \tan \alpha_E - \tan \alpha_w$, $\alpha_E = \arccos(r_{b1} / r_{1s})$.

To shorten the computation time, we developed a method based on blocks. With this method, the variations in profile curvature radii, maximum contact stresses and contact area width are not determined per every gear revolution (gear teeth engagement); instead, these parameters can be computed following a certain number of revolutions (block of interaction). In every following block, these variations are taken into account in accordance with Equation (8); having determined the value of ρ_{kjh} , we can move on to compute required parameters and linear tooth wear.

The variable radii of curvature ρ_{kjh} are calculated by the formula [5]:

$$\rho_{kjh} = \rho_{kj} + E_k \sum_{B_1}^{B_{\max}} D_{kjB} K_{kjB}^{-1}, \quad k = 1; 2 \quad (8)$$

where: $k = 1; 2$ are the numbers of the gears (1 – pinion, 2 – gear); B denotes the number of gear revolutions (i.e. the block range of teeth interaction) when the contact conditions are maintained constant; the range of a block can be selected in the following way: $B = 1$ revolution (accurate solution), $B = n_1$ (rev/min), $B = n_1$ (rev/hr), $B = n_1$ (revolutions per 10 hours), and so on; B_1 and B_{\max} are the first and last computational blocks, respectively; E_k is the dimensionless constant which depends on the maximum allowable gear tooth wear h_{k*} ; $D_{kjB} = K_{kjB}^2$ denotes the constant, i.e. the value which remains the same in one block, but changes in every other block.

The variation in the gear tooth profile curvature due to wear for every single block of interaction is:

$$K_{kjB} = 8 \sum^n h'_{kjn} / l_{kj}^2, \quad (9)$$

where $n = n_k = 1, 2, 3, \dots$ denotes the number of gear revolutions.

A single linear wear h'_{kjh} of the teeth at any j -th point of their profile is calculated after every block of revolutions in the time t'_{jh} ; importantly, this value cannot be accumulated. The width of the contact area $2b_{jh}$ is measured at the revolution $n_k - 1$ or at the block $B - 1$ according to (1). Hence

$$h'_{kjh} = \frac{v_j t'_{jh} (fp_{jh \max})^{m_k}}{C_k (0.35 R_m)^{m_k}} \quad (10)$$

where; $v_j = v$ denotes the sliding velocity at the j -th point of the tooth profile; $t'_{jh} = 2b_{jh} / v_0$ denotes the time of wear in the course of motion at the j -th point of contact along the tooth by the width of the contact zone $2b_{jh}$; $v_0 = \omega_1 r_1 \sin \alpha_t$ denotes the velocity of motion at the contact point along the tooth profile; f is the sliding friction factor; R_m is the immediate tensile strength of the material; C_k, m_k are the characteristics of wear resistance of the gear materials determined in accordance with the methodology given in [3], based on the results of experimental tribological tests.

The sliding velocity of the teeth in mesh is calculated as:

$$v_j = \omega_1 r_{b1} (tg \alpha_{t1j} - tg \alpha_{t2j}). \quad (11)$$

The length of the chord which replaces the involute between the points $j - 1, j + 1$ is expressed as:

$$l_{kj} = 2\rho_{kjh} \sin \varepsilon_{kjh} = \text{const}, \quad (12)$$

where $\varepsilon_{kjh} = S_{kj} / \rho_{kjh}$ describes the angle between the points j and $j + 1$;

$S_{kj} = \left| \frac{mz_k}{4} \left(\frac{1}{\cos^2 \alpha_{kj}} - \frac{1}{\cos^2 \alpha_{k,j+1}} \right) \cos \alpha \right|$ denotes the length of the involute between the points $j, j + 1$; α_j, α_{j+1} are the angles of engagement at selected involute points $j, j + 1$ [6].

Hence, following each interaction or interaction block, all computational parameters will be changed, i.e. $h_{1j}, h_{2j}, \rho_{1jh}, \rho_{2jh}, \rho_{jh}, p_{jh \max}, 2b_{jh}, t'_{jh}$.

For a given number of the revolutions n_{1s} of the pinion and the revolutions n_{2s} of the gear, and their corresponding number of blocks, the cumulative wear h_{1jn} and wear h_{2jn} at the j -points of contact are expressed as:

$$h_{1jn} = \sum_1^{n_{1s}} h_{1jB}, \quad h_{2jn} = \sum_1^{n_{2s}} h_{2jB}, \quad (13)$$

where $n_{2s} = n_{1s} / u$; $h_{kjB} = \sum h'_{kj}$ denotes the gear tooth wear per every block

The gear life t_{min} for the number of revolutions n_{1s} or n_{2s} is defined as:

$$t_{min} = n_{1s} / 60n_1 = n_{2s} / 60n_2. \quad (14)$$

3. NUMERICAL SOLUTION

The input data included: $z_1 = 20$; $z_2 = 80$; $m = 3$ mm; $u = 4$; $n_1 = 700$ rpm; $P = 6$ kW; $f = 0,05$; $b = 30$ mm; $\beta = 0^0$; $K_g = 1,6$. The following materials were used: the pinion was made of 38HMJA nitrided steel with 58 HRC; $R_m = 1040$ MPa, $C_1 = 3,5 \cdot 10^6$, $m_1 = 2$; the gear was made of 40H steel after bulk heat treatment with 53 HRC, $R_m = 981$ MPa, $C_2 = 0,17 \cdot 10^6$, $m_2 = 2,5$; $E = 2,1 \cdot 10^5$ MPa, $\mu = 0,3$. The lubricant was engine oil containing an anti-wear additive with a kinematic viscosity of $\nu_{+50^0} \approx 15$ cSt; $h_{ks} = 0,5$ mm; step $\Delta\varphi = 4^0$. The correction coefficients and geometrical parameters of the gear were as follows: $x_1 = -x_2 = 0; 0,2; 0,4; 0,6$; $a_w = 150$ mm. The blocks had the following ranges: $B = 750$ revolutions, $B = 42000$ revolutions, $B = 84000$ revolutions, $B = 420000$ revolutions and $B = 2100000$ revolutions.

When solving the problem, the following conditions are taken into account: the teeth are in double-single-double engagement; the dynamic character of work is defined by the dynamic factor K_g ; the process is conducted under boundary lubrication conditions.

The minimum life is defined as the shortest value of the teeth life t_j at particular j -points of their profiles.

The results of the solution for a cylindrical spur gear with profile correction (P–O) are given in the table.

Tab. 1. Results of the solution

$x_1, (-x_2)$	$B, \text{ rev}$					$\delta, \%$
	700	42000	84000	420000	2100000	
0	17608	17612	17616	17640	17660	0.30
0.2	28497	28500	28504	28520	28600	0.36
0.4	34266	34268	32272	32280	34400	0.39
0.6	31050	31052	31056	31080	31200	0.48

Note: $\delta = B_{\max} / B_{\min}$ denotes the deviation of gear life .

It was found that the deviation δ of the computational accuracy in the range between $t_{2100000}$ and t_{700} is negligible (not higher than 0.5%) for the selected range of the correction coefficients $x_1 = -x_2$. The computation time τ for the block $B = 2100000$ revolutions compared to $B = 700$ revolutions is shorter by about 2720 times. Fig. 1 shows the computation time for selected interaction block ranges (solid line) obtained using a standard personal computer.

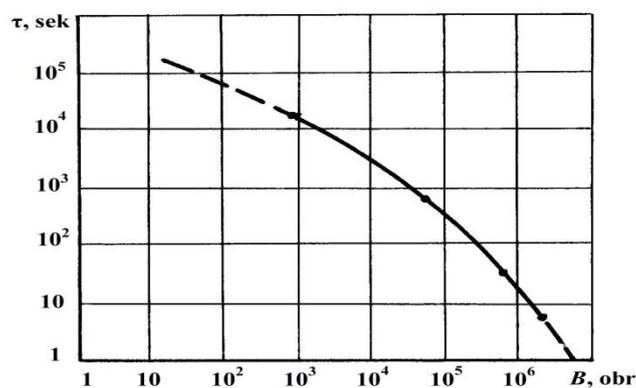


Fig. 1. Effect of block ranges on computation time [source: own study]

4. CONCLUSION

In conclusion, it has been found that increasing the block range by 3000 times, i.e. from 700 revolutions (1 min per pinion) to 2100000 revolutions (50 hrs), practically does not affect the accuracy of the results, despite the fact that the computation time is much shorter.

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