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CALCULATION METOD FOR THE EVALUATION OF INFLUENCE OF TOOTH ENGAGAMENT PARITY IN CONICAL SPUR GEAR ON CONTACT PRESSURES, WEAR AND DURABILITY

Abstract

The paper presents the results of research undertaken to determine maximum contact pressures, wear and life of conical gears, taking account gear technological correction, tooth engagement and wear-generated changes in curvature of their involute profile. The calculations were made for a reduced cylindrical gear using a method developed by authors. The effect of applied conditions of tooth engagement in the frontal and internal sections of cylindrical gear ring is shown graphically. The initial maximum contact pressures will be higher in the internal section and the highest at the entry of single tooth engagement; the increasing of correcting coefficients will cause the fall of contact and tribocontact pressures; the optimum values of correction coefficients, at which the durability of the gear will be the highest were obtained.

1. INTRODUCTION

The conic gears that allow to transmit torsion moment with the angle $90° \leq 2\delta < 90°$ between axis are widely used. In teeth interaction their two-one-two pair engagement is realized. But the methods allowing to take into account this important circumstance of corrected and uncorrected gears work, including bevel gears, are absent. For a gear with no tooth profile correction, the maximum

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contact pressures generated during tooth engagement are determined in compliance with a relative ISO standard. However, no such methods have been developed for conical gears with corrected tooth profiles. Also, the methods for assessing wear and life gears reported in the literature (Drozdov, 1975; Grib, 1982; Brauer & Andersson, 2003; Flodin & Andersson, 1997; Flodin & Andersson, 1999; Kahraman, Bajpai & Anderson, 2005; Pasta & Mariotti, 2007) can only be applied to gears with uncorrected profiles. The preliminary results of investigations conducted using methods which take into account gear tooth correction, changes in tooth profile curvature due to wear and the number of engaged tooth pairs are reported in the works (Chernets, 2013; Chernets & Chernets, 2014; Chernets & Chernets, 2015; Chernets, Yarema & Chernets, 2012; Chernets, Kelbinski & Yarema, 2011). The study below was undertaken with respect to the effect of the above factors and tooth interaction conditions on variations in the maximum contact pressures and tribotechnical parameters. The calculationfor a conical gear were made in the same way as for a reduced cylindrical gear with frontal and internal modules of conical engagement made variable over a tooth length $m_{\max} \leq m \leq m_{\min}$ (Chernets, 2013). To solve the problem, we applied methods for assessing contact strength, wear and life of spur gears.

2. METHOD FOR ESTIMATING GEAR DURABILITY

Tooth wear causes an increase in curvature radii of tooth profiles, which leads to a decrease in initial maximum contact pressures $p_{j\max}$ and contact area width $2b_j$ at every j-th point of contact. The values of $p_{jh\max}$ and $2b_{jh}$ are calculated in accordance with the modified Hertz equations:

$$p_{jh\max} = 0.564\sqrt{N'\theta/\rho_{jh}}\,,\ 2b_{jh} = 2.256\sqrt{\theta N'\rho_{jh}}\,, \tag{1}$$

where j = 0, 1, 2, 3,… are the contact points of the teeth profiles; $N' = N/w$; $N = 9550P/r_1 n_1 \cos\alpha$ is the engagement force; P is the power on the drive shaft (pinion); w is the number of engaged tooth pairs; $\theta = (1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2$; E, ν are the Young modulus and Poisson's ratios of toothed gear materials, respectively; r_1 – is the rolling radius of the pinion; r_2 is the number of revolutions of the drive shaft; $\alpha = 20°$ is the pressure angle of engaged teeth; $\rho_{jh} = \dfrac{\rho_{1jh}\rho_{2jh}}{\rho_{1jh}+\rho_{2jh}}$ is the reduced radius of curvature of the gear profile subjected to changes due to wear in a normal section; ρ_{1jh}, ρ_{jh} are the changeable radii of curvature of the pinion and gear teeth profiles, respectively.

In operation, due to the gear's wear, the initial curvature radii ρ_{1j}, ρ_{2j} (Chernets, Yarema & Chernets, 2012) of the gear profiles and the reduced curvature radius ρ_j increase.

The work (Chernets, Yarema & Chernets, 2012) presents a method which takes account of wear-generated changes in the initial radii of gear curvature in every revolution of the gear. Accordingly

$$\rho_{kjh} = \rho_{kj} + D_{jk} \sum^{n} K_{kjn}^{-1}, \; k = 1; 2, \tag{2}$$

where $n = n_k = 1, 2, 3, \ldots$ is the number of revolution of the gear; k is the numeration of gears (1 – pinion, 2 – gear); $D_{jk} = K^2_{kj}$ are the nondimensional constants at every j-th contact point depending on wear.

The wear-generated changes in gear profiles during every tooth interaction is

$$K_{kj} = 8h'_{kj} / l_{kj}^2, \tag{3}$$

where h'_{kj} in the linear wear of gear teeth at any j-th point of the profile; l_{kj} is the length of a gear chord which substitutes the involute between points j – 1, j + 1;

$$l_{kj} = 2\rho_{kjh} \sin \varepsilon_{kjh} = \text{const} \;, \tag{4}$$

where $\varepsilon_{kjh} = S_{kj} / \rho_{kjh}$ is the angle between the points j and j +1; $S_{kj} = \left| \frac{mz_k}{4} \left(\frac{1}{\cos^2 \alpha_{kj}} - \frac{1}{\cos^2 \alpha_{k,j+1}} \right) \cos\alpha \right|$ is the length of the involute between the points j, j + 1; α_j, α_{j+1} are the angles of tooth engage of selected involute points j, j + 1 (Chernets, Yarema & Chernets, 2012); m is the module of engagement; z_1, z_2 is the number of gear teeth.

To make the computation time shorter, we developed a block-based method for solving this problem. With this method, changes in profile curvature radii, reduced curvature radii and maximum contact pressures are determined following a selected number of revolutions (blocks of interactions), and not per every gear revolution (tooth engagement) as was done previously. In a block, computations are made under constant conditions of tooth engagement based on linear cumulative changes in given parameters. In a successive block of computations, the cumulative changes are taken into account after (5), (6) and then the computations are continued using new data. The changeable curvature radii ρ_{kjh} are determined as

$$\rho_{kjh} = \rho_{kj} + E_k \sum_{B_1}^{B_{\max}} D_{kjB} K_{kjB}^{-1} \;, \tag{5}$$

where B denotes the number of gear revolutions (i.e. the size of a block describing tooth interactions) with the conditions of contact maintained constant; a the size of a block can be selected as: B = 1 revolution (accurate solution), $B = n_1$ (rev/min), $B = n_1$ revolutions per one hour, $B = n_1$ revolutions per 10 hours, and so on; B_1 and $B_{\max}$ are the first and last computational block, respectively; E_k is a nondimensional constant dependent on the maximum acceptable tooth wear h_{k*}; $D_{kjB} = K^2_{kjB}$ is a constant which remains unchanged in one block but changes in every other block.

The wear-induced change in the curvature of a gear tooth profile for every single block of interaction is:

$$K_{kjB} = 8\sum^{B} h'_{kjn} / l^2_{kj}. \tag{6}$$

The unit linear tooth wear h'_{kjn} at any j-th point of gear profiles is calculated for every successive revolution of the block for the time $t'_{jh} = 2b_{kj} / \nu_0$, and it is not subject to accumulation. The width of contact area $2b_{jh}$ is determined at the revolution $n_k - 1$ or at block $B - 1$ in accordance with (1). A value of h'_{kjn} is calculated using the equation (Chernets, Yarema & Chernets, 2012):

$$h'_{kjn} = \frac{v_j t'_{jh} \left(f p_{jh\max} \right)^{m_k}}{C_k \left(0.35 R_m \right)^{m_k}}, \tag{7}$$

where t'_{jh} is the time of tooth wear at displacement along the profile of a j-th contact point over the contact area width $2b_{jh}$; $\nu_0 = \omega_1 r_1 \sin\alpha$ is the velocity of contact point travel along the tooth profile; ω_1 is the angular velocity of the pinion; ν_j is the sliding velocity at a j-th point of the gear profile; f is the sliding friction factor; $p_{jh\max}$ is the maximum tribocontact pressure (at tooth wear) at a j-th contact point; C_k, m_k are the indicators of resistance to wear of tribological pair materials; R_m the immediate tensile strength of material.

The sliding velocity of engaged teeth is calculated as:

$$v_j = \omega_1 r_{b1} \left(\tan\alpha_{1j} - \tan\alpha_{2j} \right), \tag{8}$$

where $r_{b1} = r_1 \cos\alpha$, α_{1j}, α_{2j} in compliance with (Chernets, Kelbinski & Yarema, 2011). Hence, following every interaction or a block of interaction, the parameters h_{1j}, h_{2j},, ρ_{1jh}, ρ_{2jh}, ρ_{jh}, $p_{jh\max}$, $2b_{jh}$, t'_{jh} will change.

For the applied number of pinion revolutions n_{1s} and gear revolutions n_{1s}, and the corresponding number of interaction blocks, the total tooth wear h_{1jn} and h_{2jn} at the j-th points of contact are calculated as:

$$h_{1jn} = \sum_{1}^{n_{1s}} h_{1jB}\,, \quad h_{2jn} = \sum_{1}^{n_{2s}} h_{2jB}\,, \tag{9}$$

where $n_{2s} = n_{1s} / u$; $h_{kjB} = \sum h'_{kj}$ is the tooth wear in every block of interaction; u is the gear ratio.

The service life $t_{B\min}$ of gear operation for the number of gear revolutions n_{1s} or n_{2s} is determined by:

$$t_{B\min} = n_{1s} / 60 n_1 = n_{2s} / 60 n_2\,. \tag{10}$$

The angles of transition from a double tooth engagement ($\Delta\varphi_{1F_2}$) to a single tooth engagement and, again, to a double tooth engagement ($\Delta\varphi_{1F_1}$) in a cylindrical gear with profile correction are determined in the following way:

$$\Delta\varphi_{1F_2} = \varphi_{10} - \varphi_{1F_2}\,,\ \Delta\varphi_{1F_1} = \varphi_{10} + \varphi_{1F_1}\,; \tag{11}$$

where $\varphi_{1F_2} = \tan\alpha_{F_2} - \tan\alpha$, $\varphi_{1F_1} = \tan\alpha_{F_1} - \tan\alpha$, $\varphi_{10} = \tan\alpha_{10} - \tan\alpha$,
$\tan\alpha_{F_2} = \dfrac{r_1 \sin\alpha - (p_b - e_1)}{r_1 \cos\alpha}$, $\tan\alpha_{F_1} = \dfrac{r_1 \sin\alpha - (p_b - e_2)}{r_1 \cos\alpha}$.

The angle $\Delta\varphi_{1E}$ describing the moment of teeth exit of engagement is:

$$\Delta\varphi_{1E} = \varphi_{10} + \varphi_{1E}\,, \tag{12}$$

where $\varphi_{1E} = \tan\alpha_E - \tan\alpha_t$, $\alpha_E = \arccos(r_{b1} / r_{1s})$, $r_1 = m z_1 / 2$, $p_b = \pi m \cos\alpha$,
$\tan\alpha_{10} = (1+u)\tan\alpha - \dfrac{u}{\cos\alpha}\sqrt{(r_{20}/r_2)^2 - \cos^2\alpha}$, $e_1 = \sqrt{r_{1s}^2 - r_{b1}^2} - r_1 \sin\alpha$,
$e_2 = \sqrt{r_{20}^2 - r_{b2}^2} - r_2 \sin\alpha$, $r_{20} = r_{a2} - r$, $r_2 = m z_2 / 2$, $r_{a2} = r_2 + m$, $r_{1s} = r_{a1} - r$, $r_{b1} = r_1 \cos\alpha$, $r_{b2} = r_2 \cos\alpha$, $r = 0{,}2m$; r_2 – is the radius of a pitch circle of the gear; p_b is the pitch of teeth; $u = u_K^2$ is the gear ratio of a reduced cylindrical gear; $z_1 = z_{1K} / \cos\delta_1$, $z_2 = z_{2K} / \cos\delta_2$ are the numbers of teeth in reduced cylindrical gears.

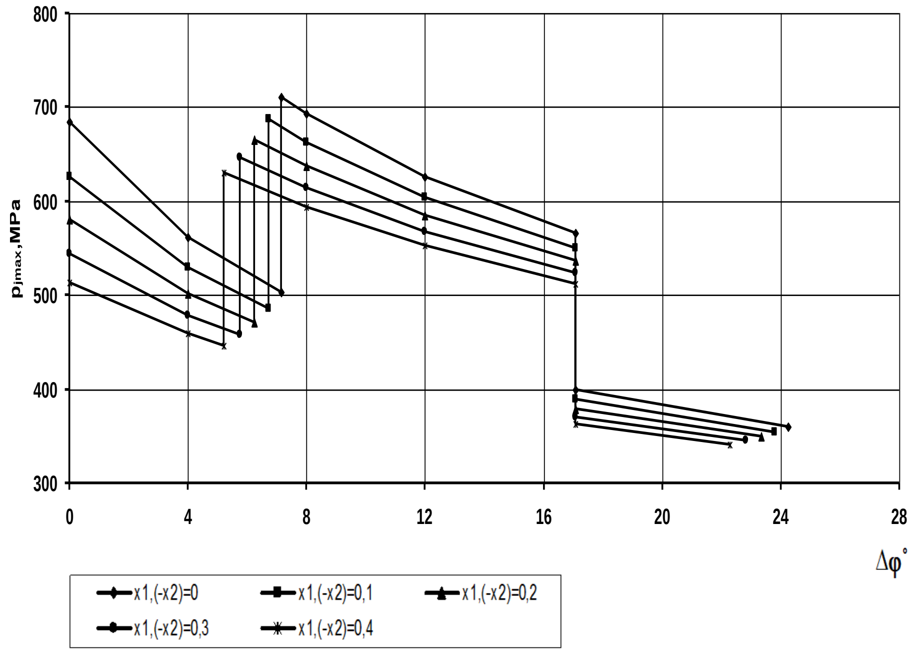
3. NUMERICAL SOLUTION

Numerical solution of the problem is made for the data: z_{1K} = 20; u_K = 3; n_1 = 750 rpm; P = 20 kW; b = 50 mm – ring width; $m_{\max}$ = 5 mm – a normal module of tooth engagement in the frontal section; $m_{\min}$ – 3.391 mm – a normal module of tooth engagement in the internal section; $\Delta\varphi$ = 4° – the increment in the pinion's angle of rotation; h_{k*} = 0,5 mm – maximum acceptable wear of gear teeth; B = 900000 revolutions. We applied boundary lubrication with a sliding friction factor set to f = 0.05. The applied technological correction coefficients were: $x_1 = -x_2$ = 0; 0,1; 0,2; 0,3; 0,4. It appears by a double-single-double tooth engagement.

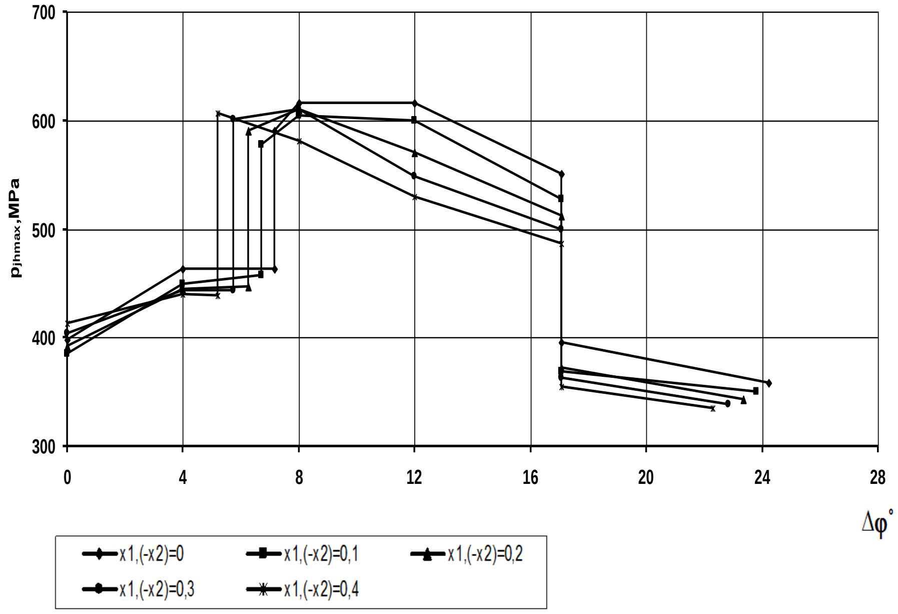
The gears were ascribed the following material properties: the pinion was made of 38HMJA steel after nitriding at a depth ranging from 0.4 mm to 0.5 mm described by 58 HRC, R_m= 1040 MPa, $C_1 = 3{,}5\cdot10^6$, m = 2; the gear was made of bulk hardened 40H steel with 53 HRC, R_m = 981 MPa, $C_2 = 0.17\cdot10^6$, m_2 = 2.5; $E = 2.1\cdot10^5$ MPa, μ = 0.3.

The results are illustrated in the figures below. Fig. 1a shows the diagram that presents changes of initial contact pressures $\boldsymbol{p_{j\max}}$ in the frontal section (external), and Fig. 1b presents their transformations $\boldsymbol{p_{j\max}}$ as a result of tooth wear up to maximum values. Respectively Fig. 2 shows contact and tribocontact pressures in the internal section.

The left- and right-hand sides of the figures show double tooth engagement, while single tooth engagement can be observed in the centre of the figures. An increase in the displacement coefficients leads to decreasing $p_{j\max}$, this decrease is particularly significant on the left. The decrease in the tribocontact pressures $p_{jh\max}$ This value is remarkably high in the entire left-hand zone of double tooth engagement and quite visible at the beginning of the double tooth engagement zone.



a)



b)

Fig. 1. Contact and tribocontact pressures in the frontal section (own study)

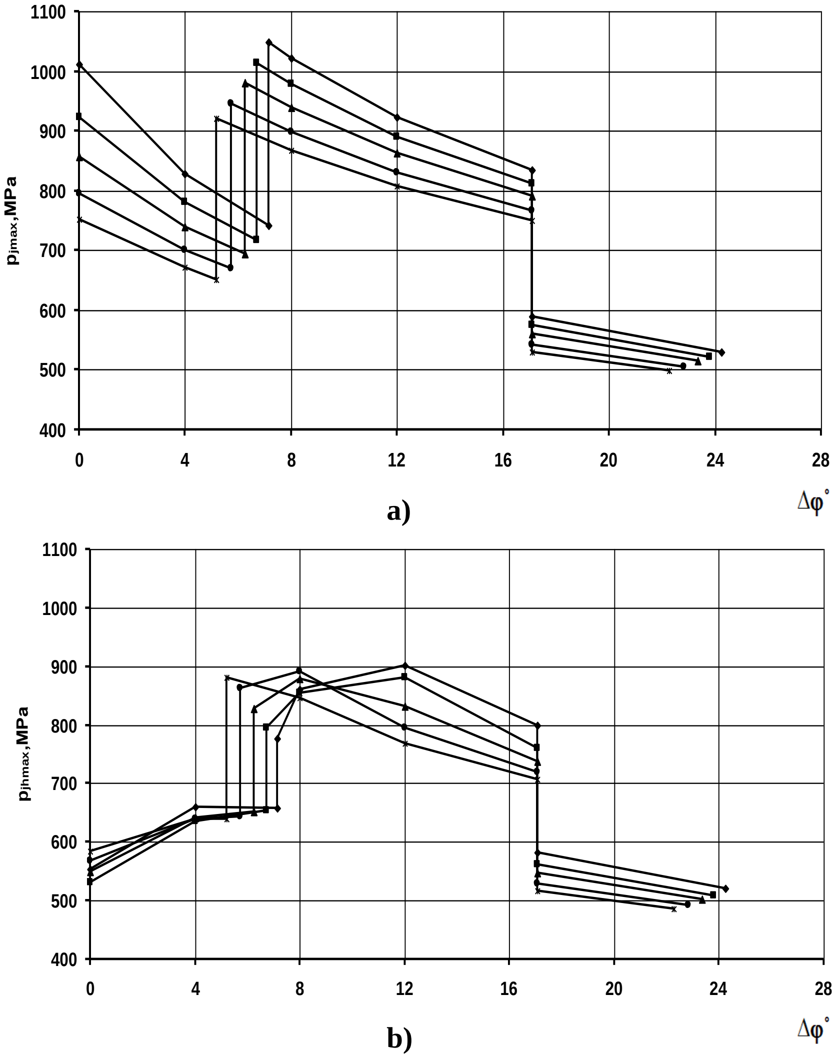


Fig. 2. Contact and tribocontact pressures in the internal section (own study)

As for this section, p_{jmax} is approx. 1.48 times higher than that in the frontal section. The change in p_{jmax} is similar to one given above (Fig. 1b).

Fig. 3 shows the diagrams of linear wear of gear profiles in the engagement zone in the internal section.

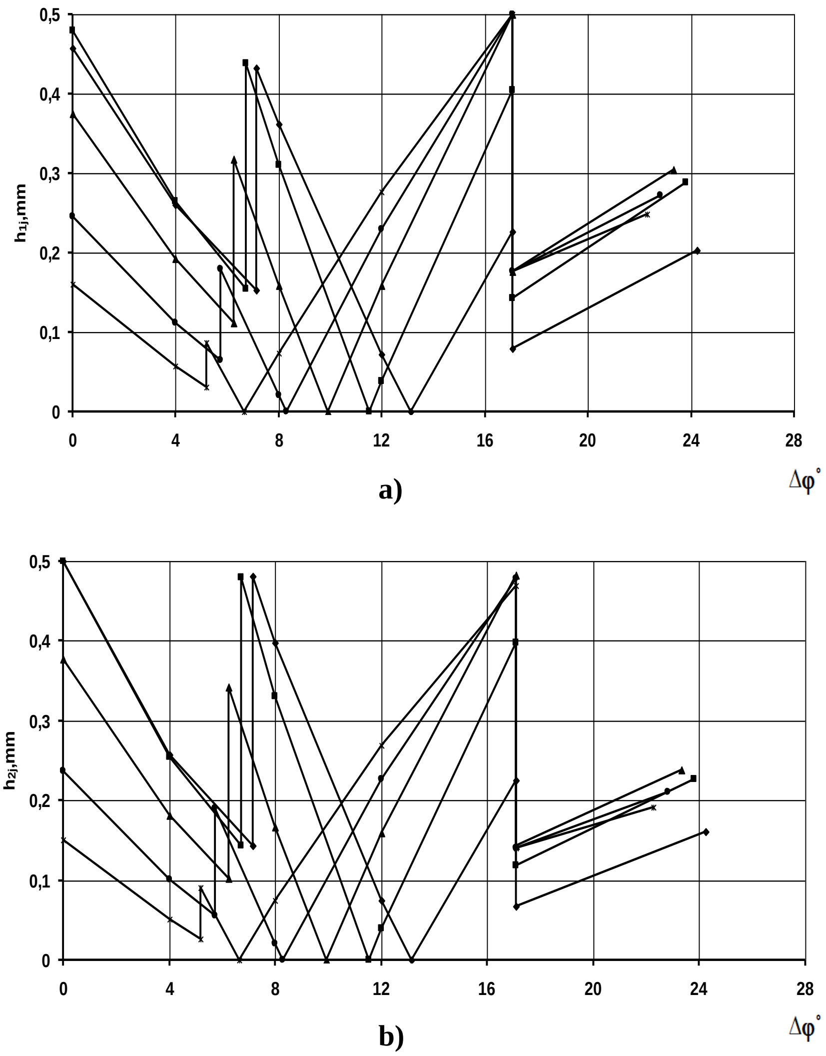


Fig. 3. Linear wear of corrected teeth: a) pinion, b) gear (own study)

Depending on a value of $x_1= -x_2$, the maximum acceptable wear of cylindrical gear teeth occurs at the entry of the left-hand zone of double tooth engagement ($x_1 = -x_2 = 0$; 0.1) and at the exit of single tooth engagement ($x_1 = -x_2 > 0.1$).

Fig. 4 illustrates the effect of the minimal life of gears (gear tooth profile points where h_{2*} is attained) on the displacement coefficients $x_1 = -x_2$.

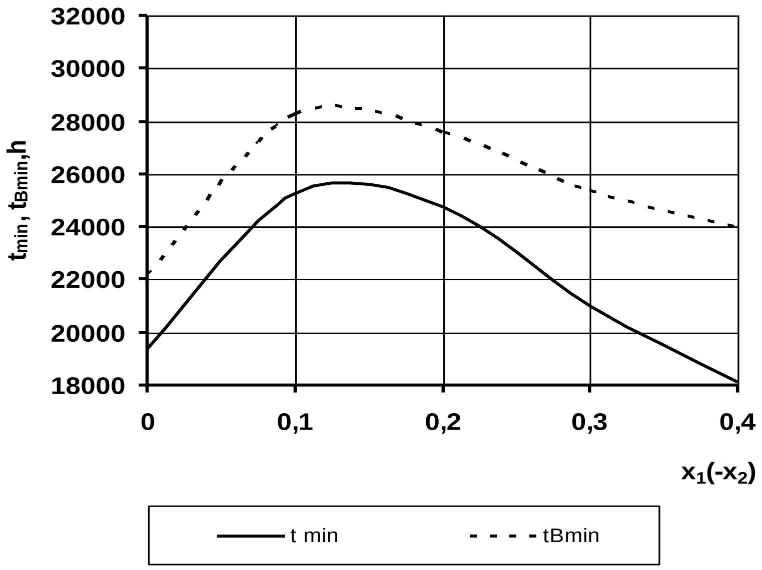


Fig. 4. Gear life: t_{min} when $p_{j\max}$ = const, $t_{B\min}$ when $p_{jh\max}$ = var (own study)

Accordingly, the gear life $t_{B\min}$ takes into account changes in $p_{j\max}$ due to tooth wear, while t_{min} is described by the assumption that $p_{j\max}$ remains constant. For a selected range of change in the displacement coefficients, the optimal gear life is at $x_1 = -x_2 \approx 0.13$.

4. CONCLUSION

The results have demonstrated that:

1. In spur gears with double-single-double tooth engagement, the initial contact pressures $p_{j\max}$ will be higher in the internal section of the gear by 1.475 times than in the frontal section due to a decrease in the module. That means that increasing of $p_{j\max}$ is proportional to decreasing of the module's value.
2. The highest values of $p_{j\max}$ can be observed at the entry of single tooth engagement in both sections.
3. The regularities concerning a decrease in $p_{j\max}$ and $p_{jh\max}$ in the left-hand zone of engagement can similarly be observed with increasing the displacement coefficients $x_1 = -x_2$.
4. We determined optimal displacement coefficients for spur gears $x_1 = -x_2 = 0.13$ which will produce the highest possible gear life.
5. The linear wear h_{1j}, h_{2j} of gear teeth in the internal section are alike.
6. Spur gears have the lowest acceptable life in their internal section.

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