

*ear-clipping, figure recognition, calculation of missing dimensions,
undirected consistent and cyclical graph, 3D visualization*

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AUTOMATION OF ROOF AREA CALCULATION PROCESS

Abstract

The paper presents approach that allows to automate roof area calculation process as well as other accompanying actions. These actions include: figure recognition, calculation of necessary dimensions, recognition of connections between roof surfaces, visualization of the roof in three-dimensional space and calculation of the roof area based on processed data. The aim of proposed approach is to automate as much actions as it is possible to reduce calculation errors and moreover increase the efficiency of whole process. Presented idea has been implemented and then tested for accuracy of calculations and capabilities. Performed tests showed that presented method is flexible and gives relatively small error in case of roof area calculation.

1. INTRODUCTION AND MOTIVATION

Nowadays many roofing companies employ person who is responsible for pricing roof plans. This pricing is based on architectural project, that is delivered by customer, which shows projection of the roof. To calculate the number of needed tiles it is necessary to calculate roof area. This process is preceded by a series of other steps – all of them done without using the computer. That kind of approach takes much time and is not efficient enough. Using the computer to automate this process would result in considerable economic and time benefits. It would also eliminate human errors and increase accuracy of calculations. Author has analyzed whole process in one of the companies in Szczecin and based on gathered observations has created new solution which is presented in following part of this paper.

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2. COMPUTATIONAL ALGORITHMS

Proposed approach to solve the problem of calculating roof area consists of ten steps. Those steps are shown in the figure 1 and discussed in further part of this paper. Solution assumes that basic data about the architectural design of the roof, like the shape of the roof surface or slope angle, and at least one length of the line segments, are entered by the human. All further steps could be automated and executed by the computer without user supervision.

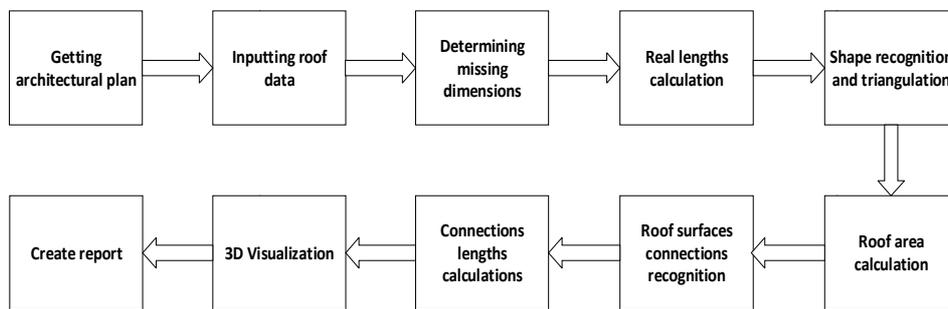


Fig. 1. Steps in proposed solution [source: own study]

2.1. Data collecting

The main part of this step is to get an architectural plan and create the model of the roof. It is created by user by drawing roof surfaces. This model should be useful for further analyzing by the computer. Necessary information that is required in further parts of presented idea are: slope angle as well as base line for each roof surface and at least one length of line segment.

Base line in figure is a line segment which is situated the lowest and is real length – not the slope line.

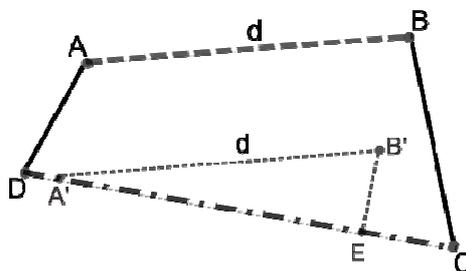


Fig. 2. Real line segment's lengths calculation [source: own study]

2.2. Getting line segments data

To calculate the roof area it is requisite to know real measurement of each line segment – not its projection. To accomplish this task first it is needed to find all missing measurements on the architectural plan and then calculate their lengths in three-dimensional space.

To calculate missing lengths it is necessary to know at least one measurement and then designate the scale for this one specific project. With this information it is possible to move measurements from local coordinate system to real one. When all dimensions are known, the real lengths of line segments in 3D can be calculated. To do that slope angle (α) and base line for each figure needs to be known. Figure 2 shows sample figure in which d is the line that length is needed to be known. First of all points A and B are projected to base line ((CD)), and then length d can be calculated from Pythagorean theorem [1] as:

$$d = \sqrt{|A'E|^2 + \left(\frac{|B'E|}{\cos \alpha}\right)^2} \quad (1)$$

2.3. Figures recognition

Next step, that needs to be done, is recognition of the shapes of each roof surface. It can be done with information that has been already collected – vertices positions. With that knowledge it is possible to recognize shape of the surface based on simple rules. Sample rules can be written as:

```
IF (number_of_vertices == 3) THEN (shape='triangle'),  
IF (number_of_vertices == 4) AND (sides_are_parallel_in_pairs) AND  
(angle_between_two_adjacent_walls != pi/2) THEN  
(shape='parallelogram').
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Similarly two other rules for rectangle and trapeze could be written.

Figures which shapes could not be recognized by presented rules (for example pentagon or deltoid) need to be triangulated – divided into $n-2$ triangles, where n is number of polygon vertices. Method that can be easily applied, in case when coordinates of vertices in polygon are already given, is triangulation by ear-clipping. This algorithm is also useful when studied figure is concave. The main idea in this method is to identify ears of polygon and their subsequent removal until remaining part of polygon is not a triangle. Ear of the polygon is a triangle formed from three consecutive vertices where two outer vertices form a diagonal. Necessary condition for created triangle to be called ear is that all diagonal must be inside the polygon [2,3]. To find it out odd parity

rule can be applied, which says when the point is placed inside or outside the polygon. When ray drawn from processed point in any direction intersects the polygon edges odd number of times then the point is placed inside the polygon, when even outside [4].

2.4. Roof area calculation

After figure shapes recognition or possible triangulation and calculating real lengths of lines segments, roof area can be calculated. Those areas are calculated using basic formulas used in math. One exception is the case of triangle – it is better to use Heron's formula rather than basic formula [1]. Heron's formula does not require additional calculations such as height of the triangle – those calculations would be the reason of greater error.

2.5. Recognition of connections between roof surfaces based on graphs

To be able to calculate the total lengths of different types of connections between roof surfaces it is necessary to recognize their types. It is possible with undirected, consistent and cyclical graph. In this case, nodes in the graph are figure's vertices and the edges are line segments. Each node has an information about its neighbors. Created graph will be traversed using modified version of breadth first search algorithm [5].

Changes in the algorithm can be presented as:

- start with queue filled with all vertices belonging to the edge of the roof, instead of one vertex,
- due to relatively small number of vertices, it is suggested to use an array of size n, where n is a number of vertices, instead of the queue,
- during graph traversal, instead of marking vertex as visited, number describing how far from the roof edge is placed is assigned to him.

Described algorithm presented as flowchart is shown on figure 3. Sample graph that was created using presented algorithm is shown on figure 4.

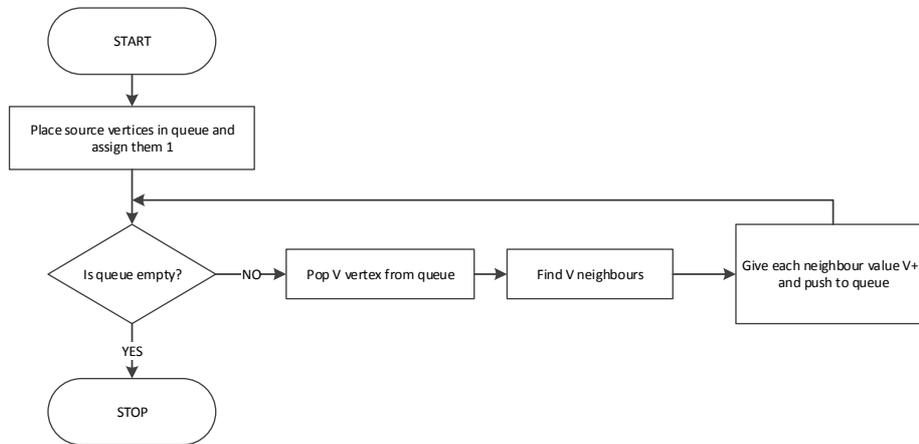


Fig. 3. Modified BFS algorithm [source: own study]

After the types are assigned to all vertices, it is possible to begin recognition of the connections between surfaces. Types that could be recognized are: valley, ridge/hip or roof edge. Beside the information about type of the vertices it is also needed to know slope angle.

There are two possible scenarios:

- line segment consists of two vertices of the same type,
- line segment consists of two vertices of different types.

In first scenario, type of connections could be determined using information about slope angles of the figures that contain examined line segment. In second, it is necessary to create three vectors and then calculate angles between them.

Vectors that need to be formed are:

- vector that describes processed line segment,
- vector that is between the lowest point of processed line segment and its neighbor situated on the same level – the same type,
- vector that is between the lowest point of processed line segment and its other neighbor, that is the same type.

Example vectors that need to be created are presented in figure 5.

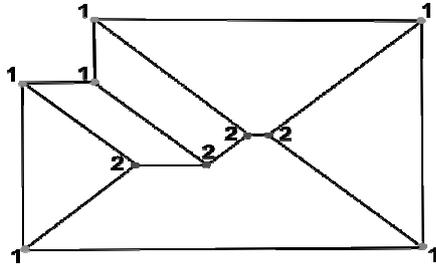


Fig. 4. Sample graph with vertices types [source: own study]

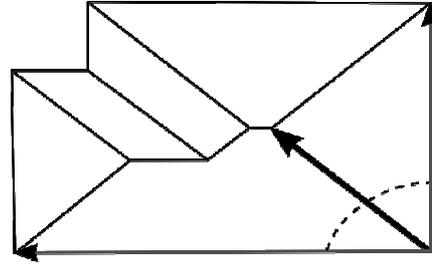


Fig. 5. Vectors creation [source: own study]

After creating said vectors, angles between examined line segment and other two created vectors are summed. The result of that operation determines type of connection. The algorithm that is used to determine type of connection between roof surfaces and edges is described using flowchart in figure 6.

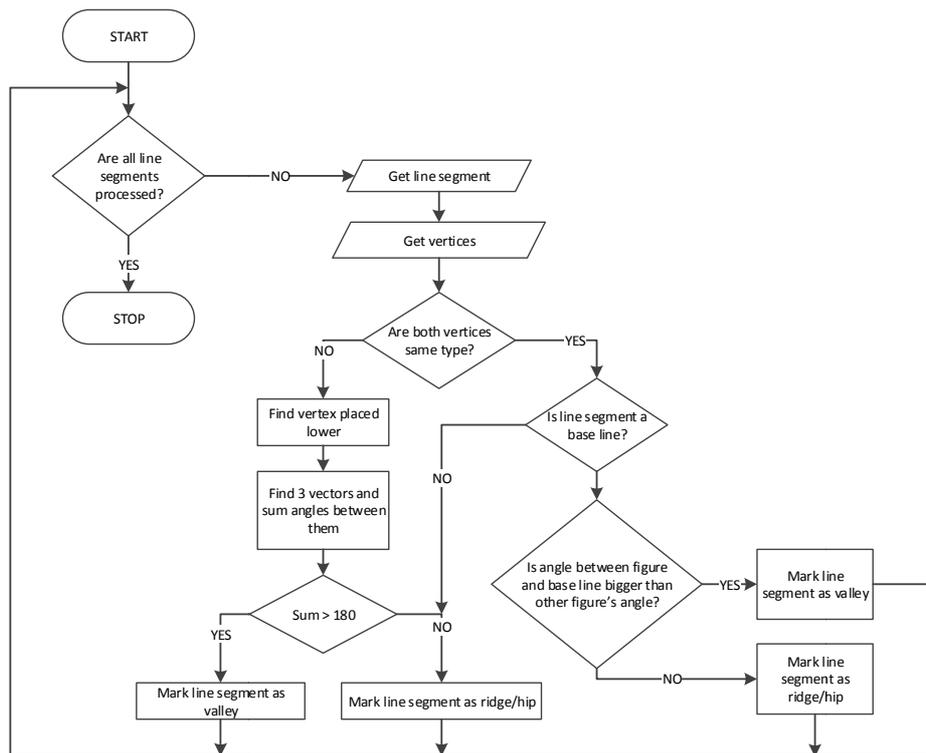


Fig. 6. Connection type recognition solution [source: own study]

2.6. Visualization of roof shape in three-dimensional space

Before roof surfaces shapes can be visualized in three-dimensional space, it is necessary to use graph, that was created already, again. When the graph was being created, each vertex had value 0 assigned to its height. Traversing the graph again, we can determine if height of each vertex is 0, or if it needs to be calculated. Algorithm that describes process of creating 3D model of the roof is shown in the figure 7.

The method of calculating height of the nodes that do not belong to base line is based on projecting processed vertex on the base line, calculate distance d between processed vertex and its projection. Then using trigonometric function [1] height can be calculated as:

$$h = \tan \alpha \cdot d \quad (2)$$

where: α – angle slope to which processed point belongs.

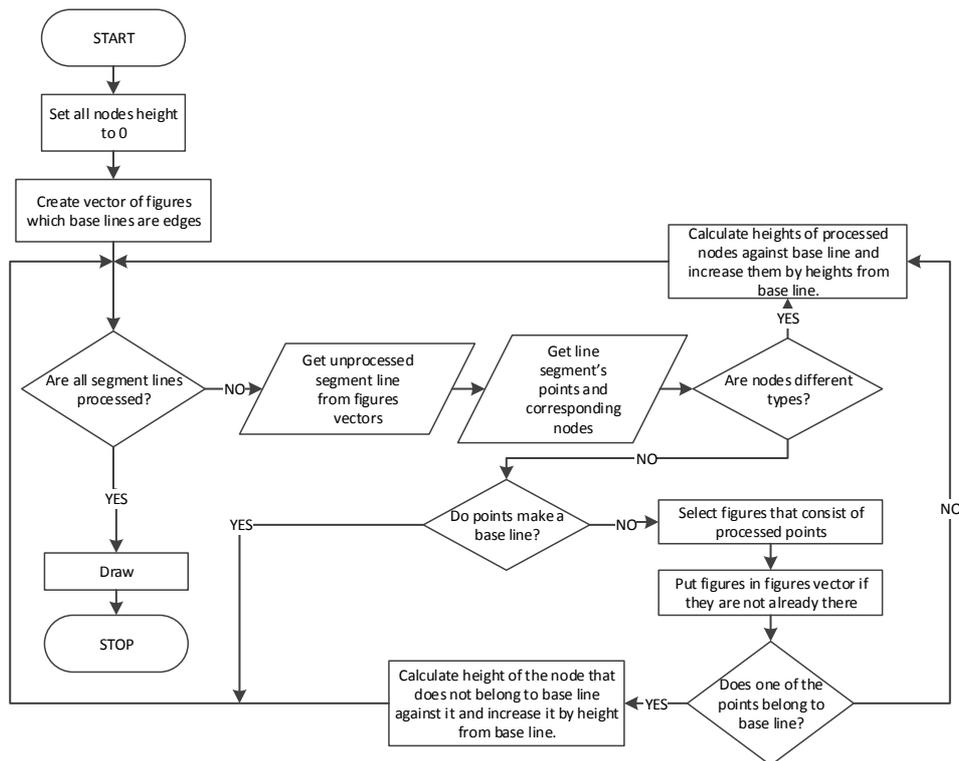


Fig. 7. 3D model creation [source: own study]

3. TESTING PROCEDURE AND RESULTS

Proposed solution, that solves the problem, has been implemented in Matlab [6] and tested for:

- errors in roof area calculation,
- correctness of connections between roof surfaces recognition,
- correctness of 3D visualization.

Tests were conducted for three sample architectural projects: hip roof, gable roof with outhouse and roof with six surfaces. Projections of these roofs are shown on figure 8. To determine the accuracy while calculating roof area relative error concept have been used:

$$\delta x = \frac{|x-x_0|}{|x_0|} \cdot 100\% \quad [7] \quad (3)$$

Main purpose was to determine how number of lengths, that are known at the beginning (does not have to be determined by computer), affects the accuracy of calculated area. Graph in figure 9 shows relative mean error for all tested projects against number of specified lengths. As can be seen, with increasing the number of lengths, relative mean error is decreasing. It is related to the fact that the less known data means that more needs to be approximated what could cause errors.

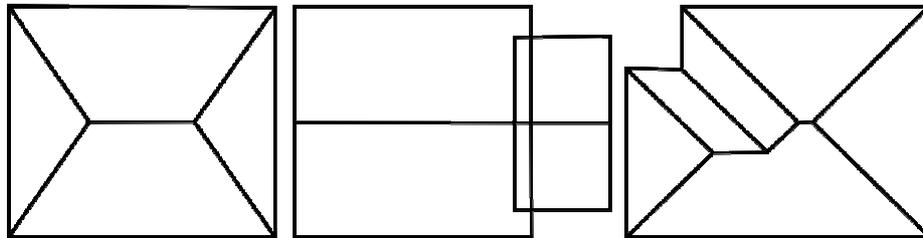


Fig. 8. Architectural plans: hip roof, gable roof with outhouse, roof with six surfaces [source: own study]

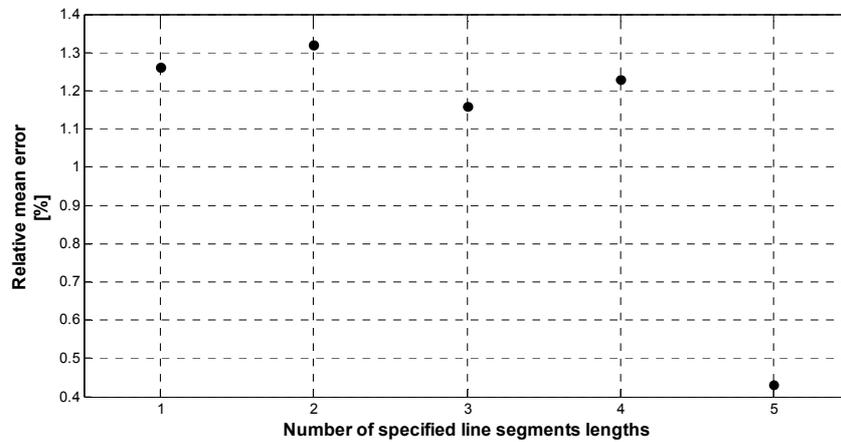


Fig. 9. Dependencies between relative mean error and number of specified data [source: own study]

Figure 10 shows recognized connections between roof surfaces. Solid lines indicate roof edges, the brightest one ridge/hip and the other color means that the line segment is a valley. In all three tested roofs, with the help of proposed algorithm it was possible to correctly identify types of connections.

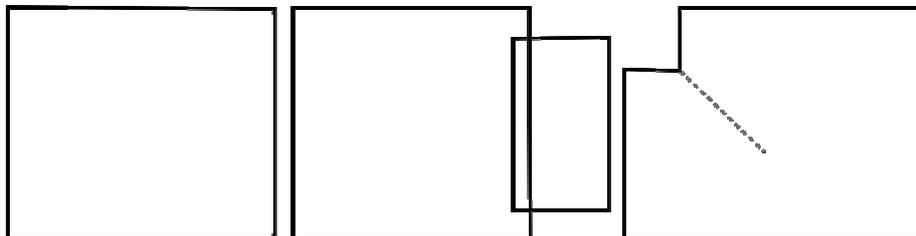


Fig. 10. Recognized connections [source: own study]

Figure 11 shows three-dimensional visualization of two tested roofs: roof with six surfaces and hip roof. Those models were created based on collected data and knowledge about architectural design such as base lines and slope angles. As it can be seen, proposed solution works well allowing to draw roof surfaces shapes in 3d.

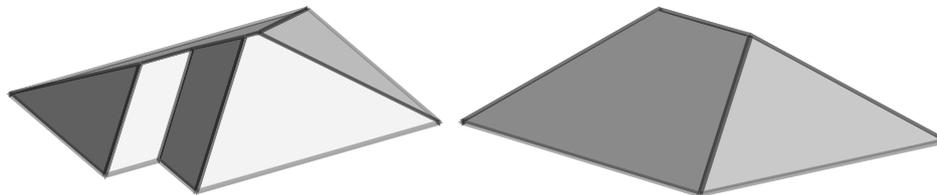


Fig. 11. 3D roof shape visualization [source: own study]

Further tests were designed to compare the effectiveness of proposed solution against two chosen free applications available online: calculator from company CB S.A. [8] and application from TONDACH [9]. Two criteria were taken into account. First of them – calculation – determines whether the solution allows to calculate roof area in specific cases. Both of the free applications allow only to calculate roof area of predefined types – they do not allow user to specify his own, custom project. Second criterion was the whether additional calculations are required before two applications or proposed solution may proceed to calculate the area. Both free applications require the user to provide real lengths of line segments – not its projection – which are given on the architectural plan. Proposed solution is more versatile, it is not needed to specify any lengths that are not given on plan. Table 1 shows comparison of proposed solution and two free applications.

Tab. 1. Comparison of two free applications CB S.A., TONDACH and proposed solution capabilities. Roof types: A – shed roof, B – gable roof, C – hip roof, D – gable roof with outhouse, E – gambrel roof, F – roof with six surfaces

	CB S.A.		TONDACH		Proposed solution	
	calculation	requires additional calculation	calculation	requires additional calculation	calculation	requires additional calculation
A	+	+	+	+	+	-
B	+	+	+	+	+	-
C	+	+	+	+	+	-
D	+	+	+	+	+	-
E	+	+	-	X	+	-
F	-	X	-	X	+	-

4. SUMMARY

Paper presents the solution of presented problem that allows to automate roof area calculation process in substantial degree. All the steps, beside entering information about roof digital model, have been automated and those are for example: recognition of connections between roof surfaces, shape of surfaces recognition and 3D visualization. Main advantages of proposed solution are that it deals with incomplete information about roof surface and that it is possible for user to provide own architectural plans instead of using only predefined ones. Based on performed tests it can be said without hesitation that proposed solution can be used to solve described problem. Considering that roofing companies sell tiles with some excess, mean relative error on 1.02% level is not a problem.

To make presented idea even more universal, new functions, like calculating area of roof surfaces that are not flat, have to be added.

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